B.1.2018 7 I lecture 7

il s is

Defil P =(A.AIR) is an equivalence relation

reflexive, transitive, symmetric

Defil ĭ C P(A) 1284 is a spartition of A if I UB FA.

IB, B'ER, B+B's BnB=0

— Ну е в зе

. xce,

efactor).

Propi. Yp g is an equivalence on A, then the guotient at Apa hf<x> (xen 4 where f<x>-hyen l xfye/

the class of is a partition of S

modulo p Prop 2 jf ù is a partition of A, then the relation

Py = (A,A,Ry) on A, where for x,yea

| YS y y cat I BeY I is an equivalence rel on A

ПА. x,4 еті)

Moreoveri Sapp=s

and Alex R

[Del x. det fiA B be a function. The Revernel of f is

the following sel om Ai.

Hx yea I x herfy as fx) = fly) 7

Propel 1) Perf is an equivalence nel an Al

2) loe have alread-24"(b) I be ym f4. prod! (1) (R) UxEA xhenf. x = f(x) = f(x) true. (T) Det x,y, 2 EA at, xherfy and y herra.

Then. FX)= fly) and fly=f(2) = f(x) = f(2). => x kerf z. (5) Let x y EA at, xhepy. Them: f(x)=fly) -s fly)-fli.

y Resfx..

(2). We know that Alteerak <h (heuf) <x>\xe A4. We only need to prove that fon xEA, if b- f(x) "mf, them (kerf)(x) = f(b).

Indeed. , det yet. We have iye (forf)(x) <=). <=> x Reef y fx) = f(y) <= f(y)=b <->ye 4-1/5).

Def 2.1 Let p be an equivalen relation om A.

The function Pp: A Alp Pp(x) = S(x)

Levery elem. it's send to its class). is called the canonical projection. Pop2! Let I be an equivalen mal. om A. Then I

1) the can projection g is sujective 2) ker Pg -s.

pool 1) Let Sexye A/p where xeA.

Themi Sex>= Pp <x> e Gmf g hence Pg is suggestive.

2) Both are relations on Art

Hx, yet we have

xher be y (.pp(x) = p gly) of <x>=f<y>

(= xgy . Hence beer pg s. The I Factorization theor

i . det fiA B be a funciona

Them 3! biv peteve function Ria Proof Symf. e.t. the following diagram is commutative:

ie f=20 fo Pe

Proof I

I ? lista) where the cronical A---> Ymf. Preof is swy)

and the canonical From the poof we get

inclusion I E (ller fl<x>) = f\*

2: Ymf B, y(b)=b.

is injective. ) (proof. (!) We assume that B exists and has the claimed

property, and we show that f is unique.:

nd Let (her f)<x> EA Berd, where xEA. © By the commutativity of the diagram, we have

f(x) = (20 Po Praet 2)(x) = 2lf (PBond (x)) =

- P(( herf) <x>) sob XEA,

It (Real) <xXés uniquely determined.

parl (5) det f': Alfere - Imp; Pl(Renf }(x)) ().

i*e,*

o We nove that the definition of Ē is correct, it does not depend on the choice of representatives. Indeed, let x'et Renka be another representative in the classes of x.

Then x herf x' so f(x) = f'(X) hence FC (best) x>) Pas

o we show that I is injective :

det (Berp) <x>, (Perf)(x) =) Almere nt

p((Perf)<x>) = f(( Roelf)x)) = f(x) = f(x).

-> x freap x' => (Perif}x>- (freef ).x> Hence of is injective.

aloe show that I is aurrective: Let be ymf no BXEA et. f(x)=b. Then b=f(x).

=f((Perflex). This proves that . Im B=ymf, hence R is awy. Š Hence B is bejective.

© le shove that the diagram is commutative.

Let xEA. We have :

(20 po preenp)(x) = 2(!p berp (x)) = Exercise

piker f) 2>) = fal , hence 20.popere fa ex 77/3 | Rove that the function fi A B is injective.

kerf = 1. (the femel d p is the equality

selation on A) proof. Note that since kesf es reflexoe, loe always.

have that a c Resf.

lie x=x' x kesf x') us" Anumite fis - Borgective

Let X, x'EA o.t. x kerfi The P(x) = f(x) fim x=x' Hence herf Sla

We aorley the 1st factorisation the oven to the fonction of given below:

We apply the 1st factone

(Example)

Il

of To î

o s*o 3*

Imf=ha,b,ch CB the

image o*f* f-Ya)=21,2,34 4%(6)=24,54 7-"(c)=664

Perk

2. Imf »B Cieta) ?(y)=y (the canonical inclusion).

(the

canonical

tx) a

projection)

ove

txJc

Abert. 221,2,34,54,54,55,644.

u

mp

(the quotient ret)'

Vet P

Albert

Berf = 30,1),(2,2), (3,3), (1,2), (the kernel of f) (2,1),(1,3), (3,1),(2,3),

(3,2),(4,4), (5,5),(4,5) (54),(66).4

In this example, le lorite davon all the functions using tables

A x 1 2 3 4 5 6 Imp a f(x) | a a a bbc. Ares Prep (4) 212,34 51,2,34 $1,2,34 24,64 54,54 664

тэц $э 21

а а

ь ,

с с

*56,*

» Кең ѕt ,1,2,34 345, -- + ((Reg)<x) а